

Scaling of combustion-generated noise

By H. A. HASSAN

Department of Mechanical and Aerospace Engineering,
North Carolina State University, Raleigh

(Received 4 March 1974)

Using the framework of Lighthill's acoustic analogy, a theory has been developed for combustion-generated noise. The theory is used to derive the scaling laws for combustion-generated noise in open turbulent flames. It is shown that the scaling laws for both power and frequency are in good agreement with experiment.

1. Introduction

In a recent review of combustion-generated noise, Strahle (1973) stressed the need for additional theoretical and experimental efforts to explain discrepancies between theory and experiment. In particular, he indicated that, for open turbulent flames, available theory is unable to explain the exact scaling laws and frequency content of combustion-generated noise. Thus, by using the theory and non-dimensionalization procedure of Chiu & Summerfield (1973), he was able to predict the scaling and order of magnitude of the observed acoustic power. However, the same procedure resulted in a scaling law for the frequency which was not observed in experiment. Another feature of combustion noise which is not explained by theory is the fact that, for all hydrocarbon-air flames, combustion noise is a broad-band noise with a single peak in the range 300–500 Hz.

The object of this investigation is to re-examine the theory of combustion noise using the framework of Lighthill's (1952) acoustic analogy as modified by Doak (1972). The source term in the wave equation for the pressure is manipulated using appropriate thermodynamic relations and the conservation of energy and species equations. The resulting expression is used as a basis for deriving scaling laws for both acoustic power and frequency. The results of the theory are shown to be in good agreement with available measurements for open turbulent flames.

2. Expression for the pressure fluctuation

Ignoring viscous effects, Doak's (1972) modified form of the acoustic-analogy equation may be expressed as

$$\frac{1}{a_\infty^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2(\rho u_i u_j)}{\partial x_i \partial x_j} - \frac{\partial^2}{\partial t^2} \left(\rho - \frac{p}{a_\infty^2} \right) \equiv q_0, \quad (1)$$

where p is the pressure, ρ is the density, u_i is the velocity and a_∞ is the speed of sound in the quiescent fluid. For a reacting gas mixture, the density may be expressed as

$$\rho = \rho(s, p, Y_k), \quad (2)$$

where s is the entropy and Y_k is the mass fraction of species k . After some rather lengthy manipulations it is shown in the appendix, equation (A 17), that, for a perfect gas with no molecular-weight change during the chemical reaction, (2) gives

$$d\rho = \frac{1}{a^2} (dp + (\gamma - 1) \rho \sum h_k dY_k) - \frac{\rho}{C_p} \left(ds + \frac{1}{T} \sum \frac{\mu_k}{M_k} dY_k \right), \quad (3)$$

where a and C_p are the frozen speed of sound and specific heat, respectively,

$$a^2 = \left(\frac{\partial p}{\partial \rho} \right)_{s, Y_k} \equiv \gamma \frac{R_0}{M_0} T = \gamma \frac{p}{\rho}, \quad C_p = \left(\frac{\partial T}{\partial h} \right)_{p, Y_k}, \quad (4)$$

T is the temperature, h and h_k are the specific enthalpies of the mixture and species k , respectively, μ is the chemical potential, R_0 is the universal gas constant, and M_k and M_0 are the molecular weights of species k and the mixture.

Equation (3) together with the conservation of species and energy equations will be used to manipulate $\partial^2(\rho - p/a_\infty^2)/\partial t^2$. Thus, from (3)

$$\frac{\partial}{\partial t} \left(\rho - \frac{p}{a_\infty^2} \right) = \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial p}{\partial t} + \frac{\gamma - 1}{a^2} \rho \sum h_k \frac{\partial Y_k}{\partial t} - \frac{\rho}{C_p} \left(\frac{\partial s}{\partial t} + \frac{1}{T} \sum \frac{\mu_k}{M_k} \frac{\partial Y_k}{\partial t} \right), \quad (5)$$

Next, (5) can be simplified by using the energy equation (Williams 1965, p. 72)

$$\frac{Ds}{Dt} + \frac{1}{T} \sum \frac{\mu_k}{M_k} \frac{DY_k}{Dt} + 0 \quad (6)$$

and the conservation of species equations

$$\rho \frac{DY_k}{Dt} = W_k, \quad (7)$$

where W_k is the production rate of species k . Thus (3) and (6) give

$$-\frac{\rho}{C_p} \left(\frac{\partial s}{\partial t} + \frac{1}{T} \sum \frac{\mu_k}{M_k} \frac{\partial Y_k}{\partial t} \right) = -u_i \frac{\partial \rho}{\partial x_i} + \frac{1}{a^2} u_i \frac{\partial p}{\partial x_i} + \frac{(\gamma - 1) \rho}{a^2} u_i \sum h_k \frac{\partial Y_k}{\partial x_i}. \quad (8)$$

Using (4), (7) and (8), (5) reduces to

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho - \frac{p}{a_\infty^2} \right) &= \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial p}{\partial t} - (\gamma - 1) \frac{\partial}{\partial x_i} \left(\frac{u_i p}{a^2} \right) + (\gamma - 1) \frac{p}{a^2} \frac{\partial u_i}{\partial x_i} - p u_i \frac{\partial}{\partial x_i} \left(\frac{1}{a^2} \right) \\ &\quad + \frac{\gamma - 1}{a^2} \sum h_k W_k, \end{aligned} \quad (9)$$

or

$$\frac{\partial}{\partial t} \left(\rho - \frac{p}{a_\infty^2} \right) = \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial p}{\partial t} - \frac{\partial}{\partial x_i} (\rho_e u_i) + \rho_e \frac{\partial u_i}{\partial x_i} - (p - p_\infty) u_i \frac{\partial}{\partial x_i} \left(\frac{1}{a^2} \right) + \frac{\gamma - 1}{a^2} \sum h_k W_k, \quad (10)$$

where

$$\rho_e = \rho - \rho_\infty - (p - p_\infty)/a^2.$$

Equation (10) gives

$$\begin{aligned} \frac{\partial^2}{\partial t^2} \left(\rho - \frac{p}{a_\infty^2} \right) &= \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial^2 p}{\partial t^2} - \frac{\partial}{\partial x_i} \left[\frac{\partial}{\partial t} (\rho_e u_i) \right] + \frac{\gamma - 1}{a^2} \frac{\partial}{\partial t} \sum h_k W_k + O(a^{-3}) \\ &\quad + \text{higher-order terms.} \end{aligned} \quad (11)$$

The ordering with respect to inverse powers of a introduced above is similar to that carried out by Morfey (1973). It is employed here because we are interested in combustion noise at low Mach numbers.

Introducing (11) into (1), the wave equation for the pressure reduces to

$$\frac{1}{a_\infty^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = q_0 \simeq - \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial^2 p}{\partial t^2} - \frac{\gamma-1}{a^2} \frac{\partial}{\partial t} \Sigma h_k W_k + \frac{\partial}{\partial x_i} \left[\frac{\partial}{\partial t} (\rho_e u_i) \right] + \frac{\partial^2 (\rho u_i u_j)}{\partial x_i \partial x_j} + O(a^{-3}) + \text{higher-order terms.} \quad (12)$$

Thus, the source term in (12) consists of a simple source resulting from combustion, together with a dipole term which may explain the weak directionality associated with combustion noise, and the usual quadrupole term.

It is to be noted that, if the heating in the system is carried out by external means such as an arc jet and not by a chemical reaction, then (6) is replaced by

$$T(Ds/Dt) = Q, \quad (13)$$

where Q is the energy added per unit mass and unit time. Because Q is related to the stagnation-temperature rise, (12) will contain a simple source term which depends on the rise in the stagnation temperature.

Formal solution of (12) yields

$$\begin{aligned} \Delta p = p - p_\infty &= \frac{1}{4\pi} \int \frac{q_0(\boldsymbol{\xi}, \eta)}{r} d\boldsymbol{\xi} \\ &\simeq \frac{1}{4\pi} \left\{ - \int \frac{1}{r} \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial^2 p}{\partial t^2} d\boldsymbol{\xi} - (\gamma-1) \int \frac{1}{a^2 r} \left[\frac{\partial}{\partial t} \Sigma h_k W_k \right] d\boldsymbol{\xi} \right. \\ &\quad \left. + \int \frac{1}{r} \frac{\partial}{\partial \xi_i} \left[\frac{\partial}{\partial t} (\rho_e u_i) \right] d\boldsymbol{\xi} + \int \frac{1}{r} \frac{\partial^2 (\rho v_i v_j)}{\partial \xi_i \partial \xi_j} d\boldsymbol{\xi} \right\}, \quad (14) \end{aligned}$$

where $\eta = t - r/a_0$, $r = |\mathbf{x} - \boldsymbol{\xi}|$. (15)

The far-field sound pressure for a compact source follows from (14) as

$$\begin{aligned} \Delta p \simeq - \frac{1}{4\pi x} \left\{ \int \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial^2 p}{\partial t^2} d\boldsymbol{\xi} + (\gamma-1) \int \frac{1}{a^2} \left[\frac{\partial}{\partial t} \Sigma h_k W_k \right] d\boldsymbol{\xi} \right. \\ \left. + \frac{1}{a_\infty} \frac{x_i}{x} \int \frac{\partial}{\partial t} (\rho_e u_i) d\boldsymbol{\xi} - \frac{1}{a_\infty^2} \frac{x_i x_j}{x^2} \int \frac{\partial^2 (\rho u_i u_j)}{\partial t^2} d\boldsymbol{\xi} \right\}. \quad (16) \end{aligned}$$

If (13) is used instead of (6), then (16) takes the form

$$\begin{aligned} \Delta p \simeq - \frac{1}{4\pi x} \left\{ \int \left(\frac{1}{a^2} - \frac{1}{a_\infty^2} \right) \frac{\partial^2 p}{\partial t^2} d\boldsymbol{\xi} - \int \frac{\partial}{\partial t} \left(\frac{\rho Q}{C_p T} \right) d\boldsymbol{\xi} \right. \\ \left. + \frac{1}{a_\infty} \frac{x_i}{x} \int \frac{\partial}{\partial t} (\rho_e u_i) d\boldsymbol{\xi} - \frac{1}{a_\infty^2} \frac{x_i x_j}{x^2} \int \frac{\partial^2 (\rho u_i u_j)}{\partial t^2} d\boldsymbol{\xi} \right\}. \quad (17) \end{aligned}$$

3. Scaling laws

As a first step in deriving appropriate scaling laws the following dimensionless quantities are introduced:

$$p' = \frac{p}{p_\infty}, \quad u'_i = \frac{u_i}{u}, \quad t' = \left(\frac{t}{l/u} \right), \quad \xi' = \frac{\xi}{l}, \quad x'_i = \frac{x_i}{R}, \quad \rho'_e = \frac{\rho_e}{\rho_\infty},$$

$$(\Sigma h_k W_k)' = \frac{\Sigma h_k W_k}{qw}, \quad (18)$$

where q is the heat released per unit mass of mixture, w is a characteristic production rate (Williams 1965, p. 98), l is a characteristic dimension, u is the mean velocity and R is a typical far-field distance. When (18) is used in (16), one finds

$$\Delta p' = \frac{\Delta p}{p_\infty} \simeq -\frac{1}{4\pi x'} \frac{l}{R} M \left\{ \int M \left[\left(\frac{a_\infty}{a} \right)^2 - 1 \right] \frac{\partial^2 p'}{\partial t'^2} d\xi' \right.$$

$$+ \lambda \int \left(\frac{a_\infty}{a} \right)^2 \frac{\partial}{\partial t'} (\Sigma h_k W_k)' d\xi' + \gamma \frac{x'_i}{x'} M^2 \int \frac{\partial^2}{\partial t'^2} (\rho'_e u'_i) d\xi'$$

$$\left. - \gamma \frac{x'_i x'_j}{x'^2} M^3 \int \frac{\partial^2}{\partial t'^2} (\rho'_e u'_i u'_j) d\xi' \right\}, \quad (19)$$

where
$$\lambda = (\gamma - 1) qw l / p_\infty a_\infty, \quad M = u/a_\infty. \quad (20)$$

Equation (19) shows that the pressure fluctuation depends on, in addition to the ratio of specific heats, which is essentially constant, the two dimensionless numbers λ and M . Therefore, from dimensional considerations (Kline 1965, chap. 1) one concludes that any dimensionless quantity characterizing combustion noise must be a function of both λ and M . Thus, if ν is the characteristic frequency, then

$$\nu l / u = G(\lambda, M). \quad (21)$$

Similarly, at low Mach numbers, where, as is seen from (19), combustion noise is dominant and thus

$$\Delta p \simeq -\frac{1}{4\pi x'} \left(\frac{l}{R} \right) M p_\infty \lambda \int \left(\frac{a_\infty}{a} \right)^2 \frac{\partial}{\partial t'} (\Sigma h_k W_k)' d\xi', \quad (22)$$

the far-field power output can be written as

$$P_S = K(\gamma - 1)^2 q^2 w^2 l^4 u^2 / 4\pi \rho_\infty a_\infty^5, \quad K = K(\lambda, M). \quad (23)$$

At higher Mach numbers other terms in (19) have to be included. In spite of this, the representation indicated in (23) for the far-field power output may be employed.

The choice of G and K is guided by experiment and, as is seen below, the expressions for the turbulent flame speed and the flame thickness. Examination of the measurements of Smith & Kilham (1963) and Shivashankara, Strahle & Handley (1973, referred to below as SSH) shows that the results of experiment are adequately correlated if one assumes that

$$\left. \begin{aligned} l &= (l_f D)^{\frac{1}{2}}, \quad G(\lambda, M) = A(\lambda/M), \\ K &= K(M) = K_0(1 + a_1 M + a_2 M^2 + \dots), \end{aligned} \right\} \quad (24)$$

where D is the burner diameter, l_f is the flame thickness and A, K_0, a_1, a_2, \dots , are constants to be determined from experiment. The choice of the expression for $G(\lambda, M)$ is prompted by the observation that the characteristic frequency is essentially the same for all hydrocarbon-air flames. Using (24), one obtains

$$\nu = (\gamma - 1) A q w / p_\infty, \quad (25)$$

$$P_S = K(M) [(\gamma - 1) q w l_f D u]^2 / 4\pi \rho_\infty a_\infty^5. \quad (26)$$

Equation (24) will be justified by showing that K is of order unity and that (25) and (26) lead to adequate scaling laws.

Using the relation (Williams 1965, p. 98)

$$w = \rho_\infty S_t / l_f, \quad (27)$$

where S_t is the turbulent flame velocity, (25) and (26) take the forms

$$\nu = (\gamma - 1) A \rho_\infty q S_t / p_\infty l_f \quad (28)$$

and

$$P_S = K \rho_\infty [(\gamma - 1) q u D S_t]^2 / 4\pi a_\infty^5. \quad (29)$$

There are a number of relations (Williams 1965, p. 168) relating S_t to S_l , the laminar flame velocity. These relations are, in general, not consistent and none of them is valid over a wide range of Reynolds numbers. This situation makes it difficult to derive scaling laws valid over a wide range of the parameters. Results based on the wrinkled-flame model show that, at low velocities, S_t is determined by S_l and the turbulence intensity, while at high turbulence intensities or high velocities, S_t is proportional to the turbulence intensity. On the other hand, in the distributed-reaction model, the relation [see (27)]

$$w = w_t = w_l \quad (30)$$

gives a relation between S_t and S_l .

4. Discussion of the results

We proceed now to show that K is of order unity and that (28) and (29) give good agreement with available experiments. Measurements of the Reynolds number and turbulence intensity and length scale were not reported in the experiments of Smith & Kilham (1963) and SSH. Thus, it is difficult to find out beforehand which expression for S_t is to be used in conjunction with (28) and (29). Lacking this information, it will be assumed, for the sake of estimating K , that the numerical value of S_t is approximately that of S_l at low velocities (of the order of 50 ft/s) and that the expression given by Bollinger & Williams (1949),

$$S_t = C_1 u^{\frac{1}{2}} D^{\frac{1}{2}} S_l, \quad C_1 = \text{constant}, \quad (31)$$

is adequate for estimating S_t at the higher velocities. Values of S_l are taken from Dugger (1952). The quantity q is related to the heat of combustion H per unit mass of fuel by the relation

$$q = [f/(1+f)] H \simeq fH \quad \text{when } f \ll 1, \quad (32)$$

where f is the fuel-to-air ratio. Values of H are taken from Keenan & Kaye (1948, p. 95).

Using the ethylene data shown in figure 5 of Smith & Kilham (1963) and assuming S_t to be 63 cm, $H = -20\,276$ B.Th.U./lb, $\rho_\infty = 1.293 \times 10^{-3}$ g/cm³, $a_\infty = 3.41 \times 10^4$ cm/s and $\gamma = 1.3$, one finds that

$$K \simeq 0.72 \quad \text{for ethylene.} \quad (33)$$

Similarly, using the propylene data of Smith & Kilham with $S_t = 42$ cm/s and $H = -19\,683$ B.Th.U./lb, one obtains

$$K \simeq 1.4 \quad \text{for propylene.} \quad (34)$$

Finally, using table 1 in SSH one finds for $S_t = 30$ cm/s, $u = 100$ ft/s and $H = -19\,929$ B.Th.U./lb

$$K \simeq 1.3 \quad \text{for propane.} \quad (35)$$

Because other terms in (19) become important at the higher velocities, some variation of K with Mach number, such as indicated in (24), is expected. Thus, the data in SSH result in $K \simeq 1.9$ for $u = 200$ ft/s and $K \simeq 3.1$ for $u = 600$ ft/s. It is seen that the above estimates support the earlier contention that K is of order unity. Also, with a value of K of order unity, (29) gives a good estimate of available acoustic-power measurements in open turbulent premixed flames.

As was pointed out by Strahle, a successful scaling law must predict the scaling of both the power output and the frequency. The characteristic frequency is given by (28), or using (32)

$$\nu = (\gamma - 1) A \rho_\infty f H S_t / p_\infty l_f. \quad (36)$$

Because H , and to a lesser extent S_t and l_f , are essentially the same for hydrocarbon fuels and hydrocarbon-air flames, one concludes from (36) that the characteristic frequency is essentially the same for hydrocarbon-air flames (SSH). Assuming that $l_f = 0.1$ cm and using the data of SSH one finds that $A \simeq -0.1$ (A is negative because, by convention, H is negative). As may be seen from (36), flames other than hydrocarbon-air flames which are characterized by different S_t and H will have different characteristic frequencies (Knott 1971).

Using the method of least squares, the measurements of SSH were correlated for lean mixtures by the expression

$$P_s = 4.89 \times 10^{-5} u^{2.68} D^{2.84} S_t^{1.35} f^{0.41} W, \quad (37)$$

where u and S_t are in ft/s and D is in ft. As was pointed out by SSH, analysis of errors due to the regression fit indicated a mean error of 5.7% with the maximum and minimum errors being 124% and -55% respectively; the standard deviation was 37%. Using (29), (31) and (32), one finds

$$P_s = K_s \frac{(\gamma - 1)^2}{4\pi a_\infty^5} \rho_\infty H^2 f^2 u^{2.5} D^3 S_t^3, \quad K_s = C_1 K. \quad (38)$$

Comparison of (38) with (37) shows that, with the exception of the dependence on the fuel-to-air ratio, the predictions of the theory are in good agreement with measurements for lean fuel mixtures.

Although (36) yields, for the values of A and l_f given above, values of the characteristic frequency in the range 300–500 Hz for hydrocarbon–air flames, the lack of an adequate correlation for l_f in terms of the flame characteristics makes it difficult to compare (36) with equation (7) in SSH, i.e.

$$\nu = 11.83u^{0.19}D^{-0.08}S_t^{0.53}f^{-0.69} \text{ Hz}, \quad (39)$$

where u and S_t are in ft/s and D is in ft. The mean error in (39) is 0.85 % and the standard deviation is 13.4 % (SSH). With S_t given by (31) and $l_f \propto D$, (36) gives

$$\nu \propto fu^{\frac{1}{2}}D^{-\frac{1}{2}}S_t, \quad (40)$$

which does not compare well with the dependance indicated in (39).

5. Concluding remarks

Using the framework of Lighthill's acoustic analogy as modified by Doak, a theory has been presented for combustion-generated noise. It was shown that, at low Mach numbers, the pressure fluctuation is a result of a simple source term resulting from combustion, together with a weak dipole term and the usual quadrupole term. The results of the theory were used to derive scaling laws for combustion-generated noise in open flames. Although the relations in (24) were not in any way the most general relations that could be used, the results of the theory provide adequate scaling laws for both power and frequency. Because the turbulent flame velocity plays an important role in the theory, it is expected that better agreement between theory and experiment could be obtained if Reynolds number, turbulence intensity and length scales were measured in conjunction with the other acoustic measurements.

The author would like to acknowledge many helpful discussions with Professor T. H. Hodgson. The work was supported, in part, by the Department of Transportation.

Appendix

From $\rho = \rho(p, s, Y_k)$ (A 1) one can write

$$d\rho = \left(\frac{\partial\rho}{\partial p}\right)_{s, Y_k} dp + \left(\frac{\partial\rho}{\partial s}\right)_{p, Y_k} ds + \sum \left(\frac{\partial\rho}{\partial Y_k}\right)_{s, p, Y_j} dY_k. \quad (A 2)$$

At constant pressure and density, (A 2) gives

$$\left(\frac{\partial\rho}{\partial s}\right)_{p, Y_k} \left(\frac{\partial s}{\partial Y_k}\right)_{p, \rho, Y_j} + \left(\frac{\partial\rho}{\partial Y_k}\right)_{s, p, Y_j} = 0, \quad (A 3)$$

thus (A 2) can be written as

$$d\rho = \left(\frac{\partial\rho}{\partial p}\right)_{s, Y_k} dp + \left(\frac{\partial\rho}{\partial s}\right)_{p, Y_k} \left[ds - \sum \left(\frac{\partial s}{\partial Y_k}\right)_{p, \rho, Y_j} dY_k \right]. \quad (A 4)$$

The quantity $(\partial\rho/\partial s)_{p, Y_k}$ can be expressed as

$$\left(\frac{\partial\rho}{\partial s}\right)_{p, Y_k} = \left(\frac{\partial\rho}{\partial T}\right)_{p, Y_k} \left(\frac{\partial T}{\partial s}\right)_{p, Y_k}. \quad (\text{A } 5)$$

Using the equation of state

$$p = R_0 \rho T \Sigma \frac{Y_k}{M_k} \equiv \frac{R_0}{M_0} \rho T, \quad (\text{A } 6)$$

where R_0 is the universal gas constant, M_0 is the molecular weight of the mixture and M_k is the molecular weight of species k , one finds

$$\left(\frac{\partial\rho}{\partial T}\right)_{p, Y_k} = -\frac{\rho}{T}. \quad (\text{A } 7)$$

Similarly, from the equation for the differential change of the specific enthalpy h (Williams 1965, p. 62)

$$dh = T ds + \frac{dp}{\rho} + \Sigma \frac{\mu_k}{M_k} dY_k, \quad (\text{A } 8)$$

where μ_k is the chemical potential, one finds

$$C_p = \left(\frac{\partial h}{\partial T}\right)_{p, Y_k} = T \left(\frac{\partial s}{\partial T}\right)_{p, Y_k}, \quad (\text{A } 9)$$

where C_p is the frozen specific heat. Thus, from (A 7) and (A 9) one obtains

$$\left(\frac{\partial\rho}{\partial s}\right)_{p, Y_k} = -\frac{\rho}{C_p}. \quad (\text{A } 10)$$

The quantity $(\partial s/\partial Y_k)_{p, \rho, Y_j}$ is considered next. From

$$ds = \left(\frac{\partial s}{\partial p}\right)_{T, Y_k} dp + \left(\frac{\partial s}{\partial T}\right)_{p, Y_k} dT + \Sigma \left(\frac{\partial s}{\partial Y_k}\right)_{p, T, Y_j} dY_k \quad (\text{A } 11)$$

one obtains

$$\begin{aligned} \left(\frac{\partial s}{\partial Y_k}\right)_{p, \rho, Y_j} &= \left(\frac{\partial s}{\partial T}\right)_{p, Y_k} \left(\frac{\partial T}{\partial Y_k}\right)_{p, \rho, Y_j} + \left(\frac{\partial s}{\partial Y_k}\right)_{p, T, Y_j} \\ &= \left(\frac{\partial s}{\partial\rho}\right)_{p, Y_k} \left(\frac{\partial\rho}{\partial T}\right)_{p, Y_k} \left(\frac{\partial T}{\partial Y_k}\right)_{p, \rho, Y_j} + \left(\frac{\partial s}{\partial Y_k}\right)_{p, T, Y_j}. \end{aligned} \quad (\text{A } 12)$$

Again, writing $d\rho$ as

$$d\rho = \left(\frac{\partial\rho}{\partial p}\right)_{T, Y_k} dp + \left(\frac{\partial\rho}{\partial T}\right)_{p, Y_k} dT + \Sigma \left(\frac{\partial\rho}{\partial Y_k}\right)_{p, T, Y_j} dY_k,$$

one obtains

$$\left(\frac{\partial\rho}{\partial T}\right)_{p, Y_k} \left(\frac{\partial T}{\partial Y_k}\right)_{p, \rho, Y_j} + \left(\frac{\partial\rho}{\partial Y_k}\right)_{p, T, Y_j} = 0. \quad (\text{A } 13)$$

Using (A 13), (A 12) takes the form

$$\left(\frac{\partial s}{\partial Y_k}\right)_{p, \rho, Y_j} = -\left(\frac{\partial s}{\partial\rho}\right)_{p, Y_k} \left(\frac{\partial\rho}{\partial Y_k}\right)_{p, T, Y_j} + \left(\frac{\partial s}{\partial Y_k}\right)_{p, T, Y_j}. \quad (\text{A } 14)$$

However, (A 8) gives

$$\left(\frac{\partial h}{\partial Y_k}\right)_{p,T,Y_j} = T \left(\frac{\partial s}{\partial Y_k}\right)_{p,T,Y_j} + \frac{\mu_k}{M_k}.$$

Therefore

$$\left(\frac{\partial s}{\partial Y_k}\right)_{p,\rho,Y_j} = \frac{C_p}{\rho} \left(\frac{\partial \rho}{\partial Y_k}\right)_{p,T,Y_j} + \frac{1}{T} \left[\left(\frac{\partial h}{\partial Y_k}\right)_{p,T,Y_j} - \frac{\mu_k}{M_k} \right]. \quad (\text{A } 15)$$

Noting that

$$h_k = \left(\frac{\partial h}{\partial Y_k}\right)_{p,T,Y_j}, \quad \left(\frac{\partial \rho}{\partial Y_k}\right)_{p,T,Y_j} = -\frac{M_0}{M_k} \rho$$

and substituting (A 15) into (A 4), one finds

$$d\rho = \frac{d\rho}{a^2} - \frac{\rho}{C_p} \left\{ ds - \Sigma \left[-C_p \frac{M_0}{M_k} + \frac{1}{T} \left(h_k - \frac{\mu_k}{M_k} \right) \right] dY_k \right\}. \quad (\text{A } 16)$$

If the molecular weight remains constant during the chemical reaction, then (A 16) reduces to

$$d\rho = \frac{1}{a^2} [dp + (\gamma - 1)\rho \Sigma h_k dY_k] - \frac{\rho}{C_p} \left[ds + \frac{1}{T} \Sigma \frac{\mu_k}{M_k} dY_k \right]. \quad (\text{A } 17)$$

Note added in proof. On the basis of additional measurements, Strahle & Shivashankara presented, at the Fifteenth International Symposium on Combustion (1974), new correlations to replace (37) and (39):

$$P_S = 3.7 \times 10^{-6} u^{2.67} S_l^{1.83} D^{2.78} f^{-0.40} W$$

and

$$\nu = 2.3 u^{0.18} S_l^{0.88} D^{-0.13} f^{-1.21} \text{ Hz}$$

The mean error in the first correlation is 6% and the standard deviation is 41% while the corresponding figures for the second correlation are 1% and 16%.

REFERENCES

- BOLLINGER, L. M. & WILLIAMS, D. T. 1949 *N.A.C.A. Rep.* no. 932.
 CHIU, H. H. & SUMMERFIELD, M. 1973 *4th Int. Coll. on Gasdynamics of Explosions and Reactive Systems*.
 DOAK, P. E. 1972 *J. Sound Vib.* **25**, 263.
 DUGGER, G. L. 1952 *N.A.C.A. Rep.* no. 1061.
 KEENAN, J. H. & KAYE, J. 1948 *Gas Tables*. Wiley.
 KLINE, S. J. 1965 *Similitude and Approximation Theory*. McGraw-Hill.
 KNOTT, P. R. 1971 *A.I.A.A. Paper*, no. 71-732.
 LIGHTHILL, M. J. 1952 *Proc. Roy. Soc. A* **211**, 564.
 MORFEY, C. L. 1973 *J. Sound Vib.* **31**, 391.
 SHIVASHANKARA, B. N., STRAHLE, W. C. & HANDLEY, J. C. 1973 *A.I.A.A. Paper*, no. 73-1025.
 SMITH, T. J. B. & KILHAM, J. S. 1963 *J. Acoust. Soc. Am.* **35**, 715.
 STRAHLE, W. C. 1973 *A.I.A.A. Paper*, no. 73-1023.
 WILLIAMS, F. A. 1965 *Combustion Theory*. Addison-Wesley.